

# The B-Meson Distribution Amplitude in QCD

V.M. BRAUN<sup>1</sup>, D.YU. IVANOV<sup>2</sup> and G.P. KORCHEMSKY<sup>3</sup>

<sup>1</sup>*Institut für Theoretische Physik, Universität Regensburg,  
D-93040 Regensburg, Germany*

<sup>2</sup>*Institute of Mathematics, 630090 Novosibirsk, Russia*

<sup>3</sup>*Laboratoire de Physique Théorique\*, Université de Paris XI,  
91405 Orsay Cédex, France*

## Abstract:

The B-meson distribution amplitude is calculated using QCD sum rules. In particular we obtain an estimate for the integral relevant to exclusive B-decays  $\lambda_B = 460 \pm 110$  MeV at the scale 1 GeV. A simple QCD-motivated parametrization of the distribution amplitude is suggested.

---

\*Unite Mixte de Recherche du CNRS (UMR 8627)

# 1 Introduction

The B-meson distribution amplitude was introduced in [1] as the direct analogue of light-cone distribution amplitudes of light mesons [2, 3, 4] in an attempt to describe generic exclusive B-decays by the contribution of the hard gluon exchange. Since then, considerable effort has been invested to understand the QCD dynamics of heavy meson decays in the heavy quark limit. The radiative decay  $B \rightarrow \gamma e \nu$  provides one with the simplest example of such processes [5]. This form factor can be calculated in terms of the B-meson distribution amplitude to one-loop accuracy [6] and arguments have been given that the corresponding factorization formula is valid to all orders in the strong coupling [7]. Similar QCD factorization formulas have also been proposed for the related processes  $B \rightarrow \gamma \gamma$  and  $B \rightarrow \gamma \ell^+ \ell^-$  [8, 9]. For weak decays involving energetic light hadrons in the final states the QCD factorization is more complex since one must isolate the end-point soft contributions in terms of additive contributions. This is a hot topic, see e.g. [10], and the results have been encouraging although, as it has been repeatedly pointed out [11, 12], the  $1/m_b$  corrections to heavy-to-light exclusive decays are most likely large and require quantitative treatment.

The B-meson distribution amplitude plays the central role in all known factorization formulas, but, surprisingly, received relatively little attention in the past. In the present work we use QCD sum rules to present a realistic model for the B-meson distribution amplitude, consistent with all QCD constraints. We also take this opportunity to clarify its theoretical status on which there has been certain confusion. The approach that we take in this paper is inspired by the classical work by Chernyak and Zhitnitsky on the QCD sum rule analysis of the distribution amplitudes of light mesons [13]. In particular we argue that the relevant matrix element of a bilocal quark-antiquark operator can be calculated by the QCD-corrected expansion at *imaginary* light-cone distances. We find that the nonperturbative corrections remain under control and present quantitative estimates for the distribution amplitude and its first inverse moment which enters decay form factors at tree level. Our results can be considered as an extension of an earlier QCD sum rule calculation by Grozin and Neubert [14] (see also [15]) with the main difference being that we calculate NLO radiative corrections to the sum rule. This is important since the true analytic structure of the distribution amplitude is only seen at this level.

The presentation is organized as follows. Sec. 2 is introductory and contains the necessary definitions. A simple model for the distribution is obtained in Sec. 3 using QCD perturbation theory and duality. The complete sum rule is constructed in Sec. 4 where we discuss the structure of nonperturbative corrections. This Section also contains our main results, the summary and conclusions.

## 2 Definitions

Following Ref. [16] we define the B-meson distribution amplitude as the renormalized matrix element of the bilocal operator built of an effective heavy quark field  $h_v(0)$  and a

light antiquark  $\bar{q}(tn)$  at a light-like separation:

$$\langle 0 | \left[ \bar{q}(tn) \not{n} [tn, 0] \Gamma h_v(0) \right]_R | \bar{B}(v) \rangle = -\frac{i}{2} F(\mu) \text{Tr} [\gamma_5 \not{n} \Gamma P_+] \Phi_+(t, \mu) \quad (1)$$

with

$$[tn, 0] \equiv \text{Pexp} \left[ ig \int_0^t du n_\mu A^\mu(utn) \right]. \quad (2)$$

Here  $v_\mu$  is the heavy quark velocity,  $n_\mu$  is the light-like vector,  $n^2 = 0$ , such that  $n \cdot v = 1$ ,  $P_+ = \frac{1}{2}(1 + \not{n})$  is the projector on upper components of the heavy quark spinor,  $\Gamma$  stands for an arbitrary Dirac structure,  $|\bar{B}(v)\rangle$  is the  $\bar{B}$ -meson state in the heavy quark effective theory (HQET) and  $F(\mu)$  is the decay constant in HQET, which is related to the physical B-meson decay constant to one-loop accuracy as

$$f_B \sqrt{m_B} = F(\mu) \left[ 1 + \frac{C_F \alpha_s}{4\pi} \left( 3 \ln \frac{m_b}{\mu} - 2 \right) + \dots \right]. \quad (3)$$

The notation  $[\dots]_R$  in (1) stands for the renormalization in a  $\overline{\text{MS}}$ -like scheme and  $\mu$  here and below refers to the  $\overline{\text{MS}}$  normalization scale.

The invariant function  $\Phi_+(t, \mu)$  where  $t$  is a real number defines what is usually called the leading twist B-meson distribution amplitude in position space, in contrast to the amplitude  $\Phi_-(t, \mu)$  which involves a different light-cone projector —  $\not{\bar{n}}$  instead of  $\not{n}$  — in between the quarks; here  $\bar{n}^2 = 0$ ,  $\bar{n} \cdot n = 2$ . This name is not exact since the translation symmetry of the theory is broken by presence of the effective heavy quark field and hence neither geometrical nor collinear twist are defined. In the present paper we only consider the distribution  $\Phi_+(t, \mu)$  and its Fourier transform

$$\begin{aligned} \phi_+(k, \mu) &= \frac{1}{2\pi} \int_{-\infty}^{\infty} dt e^{ikt} \Phi_+(t - i0, \mu), \\ \Phi_+(t, \mu) &= \int_0^{\infty} dk e^{-ikt} \phi_+(k, \mu), \end{aligned} \quad (4)$$

where in the first equation the integration contour goes below the singularities of  $\Phi_+(t, \mu)$  that are located in the upper-half plane.

The scale dependence of the distribution amplitude is driven by the renormalization of the corresponding nonlocal operator  $O_+(t) = \bar{q}(tn) \not{n} [tn, 0] \Gamma h_v(0)$ . The corresponding  $Z$ -factor was computed in [16] to one-loop order. In momentum space, the result reads

$$O_+^{\text{ren}}(k, \mu) = \int dk' Z_+(k, k'; \mu) O_+^{\text{bare}}(k'), \quad (5)$$

where

$$\begin{aligned} Z_+(k, k'; \mu) &= \delta(k - k') + \frac{\alpha_s C_F}{4\pi} Z_+^{(1)}(k, k'; \mu) + \dots, \\ Z_+^{(1)}(k, k'; \mu) &= \left( \frac{4}{\bar{\epsilon}^2} + \frac{4}{\bar{\epsilon}} \ln \frac{\mu}{k} - \frac{5}{\bar{\epsilon}} \right) \delta(k - k') - \frac{4}{\bar{\epsilon}} \left[ \frac{k}{k'} \frac{\theta(k' - k)}{k' - k} + \frac{\theta(k - k')}{k - k'} \right]_+ \end{aligned} \quad (6)$$

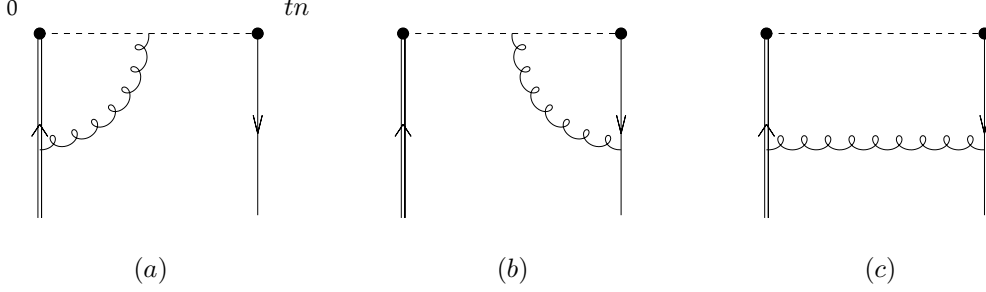


Figure 1: One-loop renormalization of the nonlocal light-cone operator built of one light and one effective heavy quark field (double line). The dashed line indicates the gluon Wilson line insertion in between the quark fields.

with  $d = 4 - \epsilon$  and the standard notation  $2/\hat{\epsilon} = 2/\epsilon - \gamma_E + \ln 4\pi$ . Here  $[\dots]_+$  is the usual “plus”-distribution.

In order to understand the meaning of this result it is instructive to consider operator renormalization in position space. The corresponding to (6) one-loop expression is

$$O_+^{\text{ren}}(t, \mu) = O_+^{\text{bare}}(t) + \frac{\alpha_s C_F}{4\pi} \left\{ \left( \frac{4}{\hat{\epsilon}^2} + \frac{4}{\hat{\epsilon}} \ln(it\mu) \right) O_+^{\text{bare}}(t) - \frac{4}{\hat{\epsilon}} \int_0^1 du \frac{u}{1-u} [O_+^{\text{bare}}(ut) - O_+^{\text{bare}}(t)] - \frac{1}{\hat{\epsilon}} O_+^{\text{bare}}(t) \right\}, \quad (7)$$

where the first two terms in curly brackets correspond to vertex-type corrections shown in Fig. 1a and Fig. 1b, respectively, (in Feynman gauge) and the third term takes into account the quark field renormalization:  $q^{\text{ren}} = Z_q^{-1/2} q^{\text{bare}}$  and  $h_v^{\text{ren}} = Z_h^{-1/2} h_v^{\text{bare}}$  with  $Z_q = 1 - (2/\hat{\epsilon})(\alpha_s C_F/4\pi)$ ,  $Z_h = 1 + (4/\hat{\epsilon})(\alpha_s C_F/4\pi)$  [17]. The exchange diagram in Fig. 1c is UV-finite and does not contribute [14, 16].

Note the following property: renormalization of the nonlocal light-cone operator  $O_+(t)$  (7) is quasilocal: it only gets mixed with itself and with operators with smaller light-cone separation. In fact the heavy quark vertex correction in Fig. 1a corresponds to a multiplicative (cusp) renormalization in coordinate space [18] while the light quark vertex correction is identical to the similar contribution to the light-quark-antiquark nonlocal operators [19]. For light quarks, this property of quasilocality guarantees existence of the Wilson short distance operator product expansion (OPE) since it implies that the “size” of the operator is not altered by the renormalization. In the present case, however, the local OPE does not exist because of the term  $\ln(it\mu)$  which is non-analytic at  $t \rightarrow 0$ . It is easy to see that this contribution arises from the term  $\sim (\mu t)^\epsilon/\epsilon^2$  in the dimensionally regularized diagram in Fig. 1a so that the answer for this diagram depends on the order of limits  $t \rightarrow 0$  and  $4 - d = \epsilon \rightarrow 0$ . We conclude that renormalization of the nonlocal light-cone operator built of one light and one effective heavy quark field does not commute with the short-distance expansion. In particular

$$[\bar{q}(tn) \not{n} [tn, 0] \Gamma h_v(0)]_R \neq \sum_{p=0}^{\infty} \frac{t^p}{p!} [\bar{q}(0) (\overleftarrow{D} \cdot n)^p h_v(0)]_R, \quad (8)$$

and the equality does not hold even as an asymptotic expansion. As a consequence, non-negative moments of the B-meson distribution amplitude  $\int dk k^p \phi_+(k, \mu)$  for  $p = 0, 1, 2, \dots$  are not related to matrix elements of local operators and in fact do not exist: It is easy to see that the logarithmic singularity of the amplitude in position space  $\Phi_+(t, \mu) \sim \ln(it)$  for  $t \rightarrow 0$  implies that the Fourier integral (4) is logarithmically divergent at  $k \rightarrow \infty$ , that is  $\phi_+(k, \mu) \sim 1/k$  for  $k \gg \mu$ , in agreement with the analysis in [16]. The analysis of moments  $\int dk k^p \phi_+(k, \mu)$  in [14, 20] tacitly assumes a different definition of the B-meson distribution amplitude, such that the nonlocal light-cone operator is defined as the generating function for renormalized local operators on the r.h.s. of (8). This implies e.g. that power divergences are subtracted. This is a different object which, most likely, does not satisfy any simple renormalization group equation and has no obvious relation to exclusive B-decays.

### 3 Sum Rules: Perturbation Theory

Aim of the present study is to suggest a realistic model of the B-meson distribution amplitude that would be consistent with all QCD constraints. To this end we evaluate the necessary B-meson matrix elements using the standard QCD sum rule approach [21]. In this section we set up the framework and present intermediate results that only include perturbation theory contributions and the assumption of duality. The complete treatment including nonperturbative corrections is presented in the next section.

To derive the sum rules we consider the following correlation functions in HQET:

$$i \int d^4x e^{-i\omega(vx)} \langle 0 | T \{ \bar{q}(0) \Gamma_1 h_v(0) \bar{h}_v(x) \Gamma_2 q(x) \} | 0 \rangle = -\frac{1}{2} \text{Tr} [\Gamma_1 P_+ \Gamma_2] \Pi(\omega) \quad (9)$$

and

$$i \int d^4x e^{-i\omega(vx)} \langle 0 | T \{ \bar{q}(tn) \not{v} \Gamma_1 [tn, 0] h_v(0) \bar{h}_v(x) \Gamma_2 q(x) \} | 0 \rangle = -\frac{1}{2} \text{Tr} [\not{v} \Gamma_1 P_+ \Gamma_2] T(t, \omega). \quad (10)$$

The correlation function  $\Pi(\omega)$  has a pole at  $\omega = \bar{\Lambda}$  where  $\bar{\Lambda} = m_B - m_b$  is the usual HQET parameter, and the residue at this pole is proportional to the HQET decay constant  $F(\mu)$ :

$$\Pi(\omega) = \frac{1}{2} F^2(\mu) \frac{1}{\bar{\Lambda} - \omega} + \text{higher resonances and continuum}. \quad (11)$$

Similarly,

$$T(t, \omega) = \frac{1}{2} F^2(\mu) \frac{1}{\bar{\Lambda} - \omega} \int_0^\infty dk e^{-ikt} \phi_+(k, \mu) + \dots \quad (12)$$

On the other hand, both correlation functions can be calculated in QCD at negative values of  $\omega$  of the order of 1 GeV in perturbation theory and taking into account nonperturbative effects induced by vacuum condensates [21]. Matching the two representations one obtains a sum rule. There are two technical details: First, one makes an assumption that contributions of the continuum and of higher resonances can be taken into account by the

restriction to the so-called duality region  $0 < s < \omega_0$  in the dispersion representation for the correlation functions, e.g.

$$\Pi(w) = \int_0^\infty \frac{ds}{s - \omega} \rho_\Pi(s) \rightarrow \int_0^{\omega_0} \frac{ds}{s - \omega} \rho_\Pi(s), \quad (13)$$

where  $\rho_\Pi(s)$  is the corresponding spectral density. The numerical value for the parameter  $\omega_0$  (called continuum threshold) is usually taken to be in the interval  $0.8 - 1.0$  GeV [22, 23, 24, 17]. Second, one makes the so-called Borel transformation

$$\int_0^{\omega_0} \frac{ds}{s - \omega} \rho_\Pi(s) \rightarrow \int_0^{\omega_0} ds e^{-s/M} \rho_\Pi(s) \quad (14)$$

introducing the variable  $M$  (Borel parameter) instead of the energy  $\omega$  in order to suppress higher-order nonperturbative corrections and minimize the dependence on the continuum model. The resulting sum rule for the correlation function  $\Pi(w)$  is well known [22, 23]:

$$\begin{aligned} \frac{1}{2} F^2(\mu) e^{-\bar{\Lambda}/M} &= \frac{N_c}{2\pi^2} \int_0^{\omega_0} ds s^2 e^{-s/M} \left[ 1 + \frac{\alpha_s}{\pi} \left( \frac{17}{3} + \frac{4\pi^2}{9} - 2 \ln \frac{2s}{\mu} \right) \right] \\ &\quad - \frac{1}{2} \langle \bar{q}q \rangle \left[ 1 + \frac{2\alpha_s}{\pi} - \frac{m_0^2}{16M^2} \right]. \end{aligned} \quad (15)$$

Here  $\alpha_s = \alpha_s(\mu)$ ,  $\langle \bar{q}q \rangle \simeq -(240 \text{ MeV})^3$  is the quark condensate and  $m_0^2$  is the ratio of the quark-gluon and quark condensates  $m_0^2 = \langle \bar{q}g(\sigma G)q \rangle / \langle \bar{q}q \rangle \simeq 0.8 \text{ GeV}^2$ . The given numbers correspond to the renormalization scale  $\mu = 1 \text{ GeV}$ . With the choice  $\bar{\Lambda} = 0.4 - 0.5 \text{ GeV}$  and  $w_0 = 0.8 - 1.0 \text{ GeV}$  the sum rule in (15) is satisfied for a wide range of values of the Borel parameter  $0.3 \text{ GeV} < M < \infty$  and is used [22, 23] to determine the B-meson decay constant  $F(\mu)$  in the heavy quark limit. In the numerical estimates in this paper we will take the “window”  $0.3 \text{ GeV} < M < 0.6 \text{ GeV}$  in which the matching is done [24] and use the value  $\alpha_s(1 \text{ GeV}) = 0.5$  ( $\Lambda_{\text{QCD}}^{(3)\text{NLO}} \simeq 360 \text{ MeV}$ ) which is consistent with the world average.

Our task in this work is to derive the similar sum rule for the correlation function  $T(t, \omega)$  defined in (10). The perturbative contributions are shown in Fig. 2. The corresponding sum rule reads, so far without nonperturbative corrections:

$$\begin{aligned} \frac{1}{2} F^2(\mu) e^{-\bar{\Lambda}/M} \phi_+(k, \mu) &= k\theta(2w_0 - k) \left[ \int_{k/2}^{\omega_0} ds e^{-s/M} \rho_<(s, k, \mu) + \int_0^{k/2} ds e^{-s/M} \rho_>(s, k, \mu) \right] \\ &\quad + k\theta(k - 2w_0) \int_0^{\omega_0} ds e^{-s/M} \rho_>(s, k, \mu) \end{aligned} \quad (16)$$

where

$$\begin{aligned} \rho_<(s, k, \mu) &= \frac{N_c}{4\pi^2} \left\{ 1 + \frac{\alpha_s C_F}{2\pi} \left[ \frac{7}{2} + \frac{7\pi^2}{24} - \ln^2 \frac{k}{\mu} - \frac{5}{2} \ln(x - 1) - (x - 1) \ln(x - 1) \right. \right. \\ &\quad \left. \left. - \frac{1}{2} \ln^2(x - 1) - 2 \ln \frac{k}{\mu} [1 + \ln(x - 1)] + x \ln x + \text{Li}_2 \left( \frac{1}{1 - x} \right) \right] \right\}, \end{aligned}$$

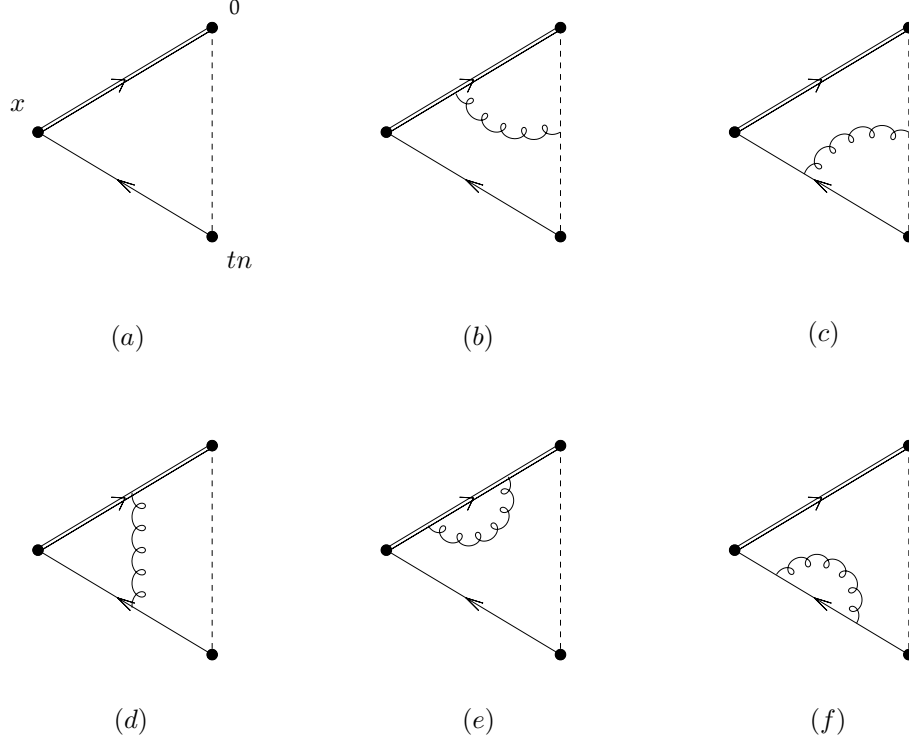


Figure 2: Correlation function (10) in QCD perturbation theory to first order.

$$\rho_{>}(s, k, \mu) = \frac{\alpha_s C_F N_c}{8\pi^3} \left[ -x + \ln(1-x) - 2(1-x) \ln(1-x) + 2 \ln^2(1-x) \right. \\ \left. + 2 \ln \frac{k}{\mu} [x + \ln(1-x)] \right]. \quad (17)$$

Here  $\text{Li}_2(x)$  is Euler dilogarithm function and we used a shorthand notation  $x = 2s/k$ ,  
Neglecting  $\alpha_s$  corrections for a moment, one gets a simple expression

$$\frac{1}{2} F^2(\mu) e^{-\bar{\Lambda}/M} \phi_+(k, \mu) = \frac{N_c}{4\pi^2} \theta(2w_0 - k) k \int_{k/2}^{\omega_0} ds e^{-s/M}. \quad (18)$$

In the local duality limit  $M \rightarrow \infty$  using the sum rule expression for  $F(\mu)$  (15) with the same accuracy,  $1/2 F^2(\mu) \simeq (N_c/6\pi^2) \omega_0^3$ , one obtains

$$\phi_+(k)^{\text{LD}} = \frac{3}{4\omega_0^3} \theta(2\omega_0 - k) k(2\omega_0 - k) \quad (19)$$

which reminds the asymptotic light-cone distribution amplitude of light mesons if rewritten in terms of the scaling variable  $\xi = k/(2\omega_0)$ . For finite values of the Borel parameter  $M$  the B-meson distribution amplitude gets skewed towards smaller values of the momentum but qualitatively remains the same, see Fig. 3. Note that it has finite support  $k < 2\omega_0$  and can be interpreted as the probability amplitude to find the light quark (on-shell) in the B-meson with momentum  $k$ .

Beyond the Born approximation this simple parton-model interpretation is lost since the distribution amplitude develops a high-momentum “tail” with  $k > 2\omega_0$  and in this

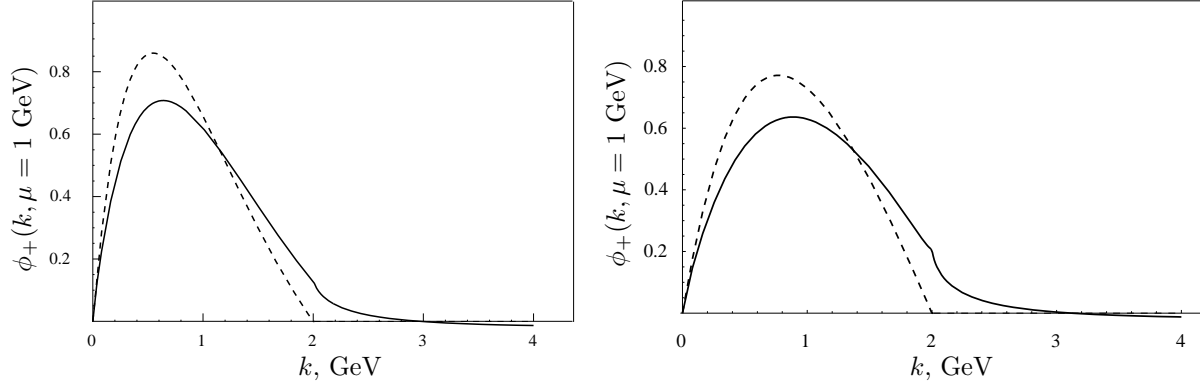


Figure 3: B-meson distribution amplitude  $\phi_+(k, \mu = 1 \text{ GeV})$  calculated from the sum rule (16) in QCD perturbation theory to leading order (dashed curves) and next-to-leading order (solid curves) for the continuum threshold  $\omega_0 = 1 \text{ GeV}$  and two values of the Borel parameter  $M = 0.3 \text{ GeV}$  (left panel) and  $M = 0.6 \text{ GeV}$  (right panel). The value of the decay constant  $F(\mu)$  appearing on the l.h.s. of the sum rule is substituted by the corresponding sum rule (15) with the appropriate accuracy (LO or NLO) and neglecting the condensate contributions.

region cannot be thought of as a probability amplitude for the two-particle state on mass shell. The  $\mathcal{O}(\alpha_s)$  radiative correction turns out to be very large ( $\sim 100\%$  of the Born term) but cancels to a large extent against the similar large radiative correction to  $F(\mu)$  [25, 22].

The numerical results for two values of the Borel parameter  $M = 0.3 \text{ GeV}$  and  $M = 0.6 \text{ GeV}$  are shown in Fig. 3. We choose  $w_0 = 1 \text{ GeV}$  for this plot and substitute the coupling  $F^2(\mu)$  appearing on the l.h.s. of the sum rule (16) by the sum rule (15) to the same accuracy, i.e. neglecting nonperturbative corrections. In this way the dependence on  $\bar{\Lambda}$  cancels out and the sensitivity to other parameters ( $\omega_0$  and  $M$ ) is strongly reduced. Indeed, it is seen in Fig. 3 that dependence on the Borel parameter is rather mild. Note that for large  $k$  the distribution amplitude becomes negative. The asymptotic behavior is

$$\phi_+(k) \sim k \text{ for } k \rightarrow 0, \quad \phi_+(k) \sim -\frac{1}{k} \ln(k/\mu) \text{ for } k \gg \mu, \quad (20)$$

in agreement with [16]. Also the scale dependence of the distribution amplitude extracted from the sum rule (16) agrees with [16]. All results are shown for  $\mu = 1 \text{ GeV}$ .

Of particular interest for the QCD description of B-decays is the value of the first negative moment

$$\lambda_B^{-1}(\mu) = \int_0^\infty \frac{dk}{k} \phi_+(k, \mu). \quad (21)$$

We obtain from the sum rules:

$$\begin{aligned} \lambda_B^{-1} &= 1.49 - 1.83 \text{ GeV}^{-1} & \text{for } \omega_0 = 1.0 \text{ GeV}, \\ \lambda_B^{-1} &= 1.79 - 2.08 \text{ GeV}^{-1} & \text{for } \omega_0 = 0.8 \text{ GeV}, \end{aligned} \quad (22)$$

where the lower value corresponds to  $M = 0.6 \text{ GeV}$  and the higher one to  $M = 0.3 \text{ GeV}$  for each choice of  $\omega_0$ . Notice that  $\lambda_B^{-1}$  decreases as  $M$  increases and in the local duality



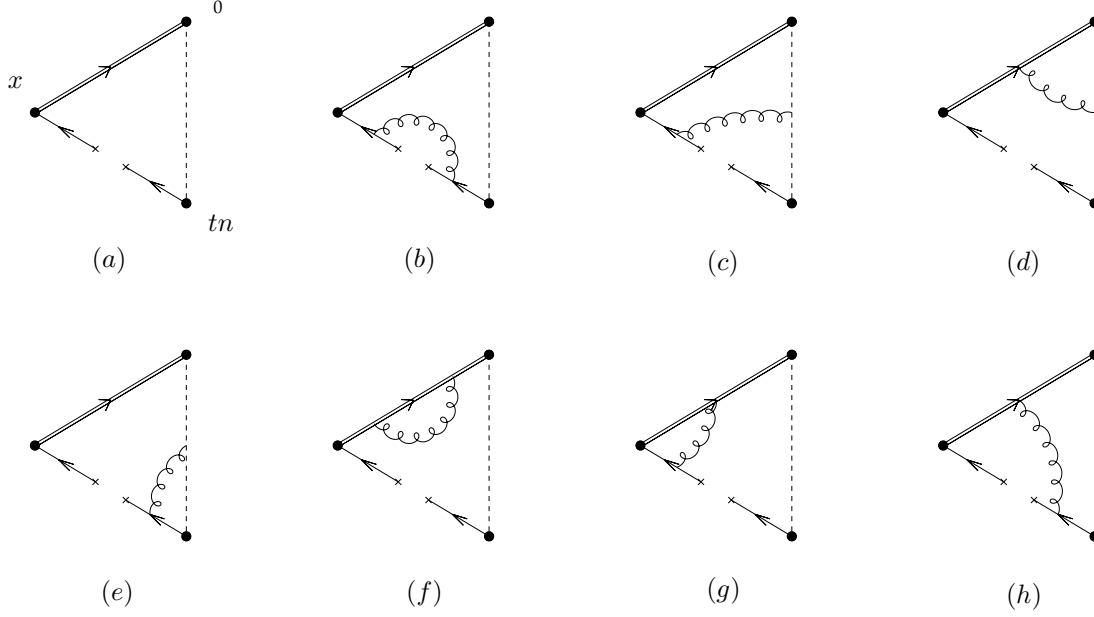


Figure 4: Quark condensate contribution to the correlation function (10).

limit we obtain

$$\lambda_B^{-1}(\mu = 2\omega_0)^{\text{LD}} = \frac{3}{2\omega_0} \left[ 1 - \frac{\alpha_s(2\omega_0)}{\pi} \left( \frac{5}{3} + \frac{5\pi^2}{36} \right) \right], \quad (23)$$

where it is taken into account that in the limit  $M \rightarrow \infty$  the sum rules effectively become normalized at the scale  $\mu = 2\omega_0$  because of subtraction of the continuum from the running coupling, cf. [26]. To avoid misunderstanding we remind that all results of this section correspond to the sum rules in QCD perturbation theory and the given numbers will be superseded by those in the next section where we consider the nonperturbative corrections.

## 4 Sum Rules: Nonperturbative Corrections

The primary source of nonperturbative corrections to the sum rules in HQET is provided by the quark condensate. The corresponding diagrams (leading and next-to-leading order) are shown in Fig. 4. The leading-order contribution in Fig. 4a is simply

$$T^{(\bar{q}q)}(t, \omega) = \frac{\langle \bar{q}q \rangle}{2\omega}. \quad (24)$$

It does not depend on the quark-antiquark separation and gives rise to the  $\delta$ -function type contribution to the r.h.s. of the sum rule in (16)

$$\dots - \frac{1}{2} \langle \bar{q}q \rangle \delta(k). \quad (25)$$

Condensates of higher dimension produce even more singular terms, the expansion goes in derivatives of the  $\delta$ -function at  $k = 0$ . This is a well-known problem which is familiar from

the QCD sum rule studies of light-cone distribution amplitudes of light mesons [13] and nucleon parton distributions [27, 28]: The short-distance OPE which is the basis of the SVZ approach is inadequate for a calculation of distribution functions point-by-point in the momentum fraction space. As a consequence, QCD sum rules cannot be used for a direct calculation of distribution amplitudes (unless they are supplemented by additional assumptions) but rather provide constraints which have to be implemented within QCD-motivated parameterizations (models) of the distributions, consistent with perturbative QCD. For a model to be self-consistent, there are three conditions:

- The end-point behavior of the distributions has to be consistent with QCD.
- The model has to be closed under the QCD evolution, i.e. calculation of the scale dependence has to be possible and not involve nonperturbative parameters other than those specified by the model at the reference scale.
- The model has to involve a minimum possible number of nonperturbative parameters.

The Chernyak-Zhitnitsky models of light-cone distribution amplitudes of light mesons give the classical example of such an approach. In this case one expands the distribution amplitude in a series over orthogonal polynomials, e.g. for the pion

$$\phi_\pi(x, \mu) = 6x(1-x) \sum_{p=0,2,\dots}^{\infty} \varphi_p(\mu) C_p^{3/2}(2x-1), \quad (26)$$

so that coefficients in this expansion correspond to (Gegenbauer) moments of  $\phi_\pi(x)$ , and defines a *model* by truncating this expansion at a certain  $p = p_{\max}$ . The first  $p_{\max}$  coefficients are then estimated using QCD sum rules. (In practice one takes  $p_{\max} = 2$  since estimates of higher-order coefficients turn out to be unreliable.) The model satisfies all the above criteria since the correct end-point behavior is built in by construction and higher-order coefficients can only get mixed with lower-order coefficients but not vice versa; it follows that the set of coefficients  $\{\varphi_0, \varphi_2, \dots, \varphi_{k_{\max}}\}$  is closed under renormalization<sup>†</sup> and the distribution amplitude  $\phi_\pi(x, \mu)$  can be calculated at arbitrary scale from its model at  $\mu = \mu_0$ . It indeed involves a minimum number of parameters, each of which has a clear meaning in QCD as the matrix element of a certain local operator and can eventually be calculated e.g. on the lattice.

In contrast to (26), the B-meson distribution amplitude cannot be written as a *sum* of independent terms that have autonomous QCD evolution but rather is given by the *integral* in the complex moments plane [16]. This feature reminds evolution of parton distributions in the deep-inelastic inclusive lepton-hadron scattering, but in difference to the latter case one cannot obtain complex moments of the B-meson distribution amplitude by analytic continuation from the set of real integers (as we mentioned in Sec. 2, every non-negative moment of  $\phi_+(k, \mu)$  diverges). As the result, one necessarily has a continuous rather than discrete set of nonperturbative parameters.

---

<sup>†</sup>The coefficients in the Gegenbauer expansion are renormalized multiplicatively to leading order because of conformal symmetry; this property is, however, not essential for our argument.

One option [28, 29, 14] is to parametrize the B-meson distribution amplitude by the matrix element of the bilocal operator in (1) at imaginary light-cone separation

$$t = -i\tau, \quad \varphi_+(\tau, \mu) = \Phi_+(-i\tau, \mu). \quad (27)$$

Obviously

$$\varphi_+(\tau, \mu) = \int_0^\infty dk \, e^{-\tau k} \phi_+(k, \mu) \quad (28)$$

and the parameter  $\lambda_B$  is given by the simple integral

$$\lambda_B^{-1}(\mu) = \int_0^\infty d\tau \, \varphi_+(\tau, \mu). \quad (29)$$

The purpose of going over to imaginary light-cone times (distances) is similar to that of the usual Wick rotation: In this way the oscillating exponents corresponding to the light-cone time dependence of intermediate states propagating along the light-cone are converted to falling exponents suppressed by the energy of the state, where the light-cone quantization is implied. Simultaneously, the normalization scale  $\mu$  acquires the physical meaning of the cutoff in energy of the intermediate states. Note that the renormalization of  $\varphi_+(\tau, \mu)$  only involves the distribution at smaller light-cone separations, cf. (7). This implies that knowledge of  $\varphi_+(\tau, \mu)$  at small distances up to  $\tau < \tau_{\max}$  is sufficient to calculate its scale dependence in the same distance range, in agreement with the self-consistency criterium formulated above.

On the other hand, it is easy to understand that the function  $\varphi_+(\tau, \mu)$  can be calculated at small  $\tau$  using OPE; expansion in vacuum condensates of increasing dimension corresponds to the Laurent expansion of  $\varphi_+(\tau, \mu)$  in powers of  $\tau$ , which is modified by calculable perturbative corrections. The condensate expansion seems to be under control up to distances of order  $\tau \sim 1 \text{ GeV}^{-1}$  (that is, of order 0.2 fm). At larger distances the OPE diverges and one has to either truncate  $\varphi_+(\tau, \mu)$  at a certain  $\tau_{\max}$  or rely on a certain model for the large  $\tau$  behaviour. Provided that the nonperturbative corrections decrease sufficiently fast for large  $\tau$  one can hope that the model assumptions do not lead to a large uncertainty in the overall result.

To illustrate this construction, we have calculated the quark condensate contribution including the  $\alpha_s$ -correction, see Fig. 4, the contribution of the gluon condensate, Fig. 5, and the contribution of the mixed condensate  $\langle \bar{q}\sigma g G q \rangle \simeq m_0^2 \langle \bar{q}q \rangle$  which is obtained as the expansion of the diagram in Fig. 4a in the background gluon field. The resulting sum rule in which we have also included the perturbative contribution of Fig. 2 reads

$$\begin{aligned} \frac{1}{2} F^2(\mu) e^{-\bar{\Lambda}/M} \varphi_+(\tau, \mu) = & \quad (30) \\ = \int_0^{\omega_0} ds \, e^{-s/M} \rho_{\text{pert}}(s, \tau, \mu) - \frac{1}{2} \langle \bar{q}q \rangle \left\{ 1 + \frac{\alpha_s C_F}{2\pi} \left[ 3 - \frac{5\pi^2}{24} - \ln^2(\tau\mu e^{\gamma_E}) - \ln(\tau\mu e^{\gamma_E}) \right. \right. \\ & \left. \left. - \ln(1 + 2\tau M) - \text{L}_2(-2\tau M) \right] \right\} + \frac{1}{48} \left\langle \frac{\alpha_s}{\pi} G^2 \right\rangle \frac{M\tau^2}{(1 + 2\tau M)^2} + \frac{1}{32} \frac{m_0^2}{M^2} \langle \bar{q}q \rangle (1 + 2\tau M), \end{aligned}$$

where the perturbative spectral density can be read off Eq. (16). The contribution of the gluon condensate turns out to be very small and will be neglected in what follows. We

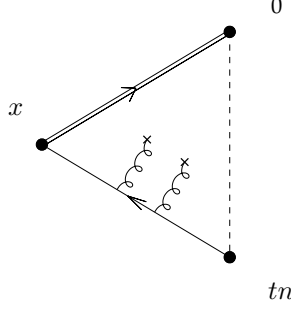


Figure 5: Gluon condensate contribution to the correlation function (10). Only this diagram contributes in the Fock-Schwinger gauge.

further note that the sum of the diagram in Fig. 4d and one half of the heavy quark self-energy correction in Fig. 4f define the universal renormalization factor of the Wilson line built of the light-like segment of length  $-i\tau$  and the time-like segment of length  $-i/M$ . It can be shown that the corresponding contributions  $\sim (\alpha_s C_F)^n$  exponentiate to all orders [30] and produce a Sudakov-like exponential suppression factor

$$S(\tau, M, \mu) = \exp \left\{ -\frac{\alpha_s C_F}{2\pi} \left[ \ln^2(\tau\mu e^{\gamma_E}) + \frac{5\pi^2}{24} - 1 - \ln \frac{\mu e^{\gamma_E}}{2M} + L_2(-2\tau M) \right] \right\} \quad (31)$$

which is the same for the quark and the quark-gluon condensate. Note that  $L_2(-2\tau M) \sim -\frac{1}{2} \ln^2(2\tau M)$  for  $\tau \gg 1/M$ . We end up with an improved sum rule

$$\begin{aligned} \frac{1}{2} F^2(\mu) e^{-\bar{\Lambda}/M} \varphi_+(\tau, \mu) = & \quad (32) \\ = \int_0^{\omega_0} ds e^{-s/M} \rho_{\text{pert}}(s, \tau, \mu) - \frac{1}{2} \langle \bar{q}q \rangle S(\tau, M, \mu) \left\{ 1 + \frac{\alpha_s C_F}{2\pi} \left[ 2 - \ln(\tau\mu e^{\gamma_E}) - \ln \frac{\mu e^{\gamma_E}}{2M} \right. \right. \\ & \left. \left. - \ln(1 + 2\tau M) \right] - \frac{1}{16} \frac{m_0^2}{M^2} (1 + 2\tau M) \right\} \end{aligned}$$

in which the double-logarithmic corrections to the quark and quark-gluon chiral condensates are resummed. We do not attempt the similar resummation in the perturbative contribution since its effect is negligible compared to the  $1/\tau^2$  falloff inherited from the Born term.

The perturbative and the nonperturbative contributions to the sum rule result (32) for  $\varphi_+(\tau, \mu)$  are shown separately as a function of distance  $\tau$  in Fig. 6 ( $\omega_0 = 1$  GeV) and Fig. 7 ( $\omega_0 = 0.8$  GeV) for two different values of the Borel parameter  $M = 0.3$  GeV and  $M = 0.6$  GeV. Note that at small distances the nonperturbative corrections are significantly smaller than the perturbative contribution. The nonperturbative correction turns to zero at a certain value of  $\tau$  as the result of the cancellation between the quark condensate contribution ( $\sim \text{const}$ ) and that of the mixed condensate ( $\sim \tau$ ). This cancellation of course cannot be taken seriously and only indicates that the OPE breaks down since the hierarchy of contributions is lost. We conclude that the classical QCD sum rule is only valid up to light-cone distances of order  $1 - 3 \text{ GeV}^{-1}$ , depending on the value of the Borel

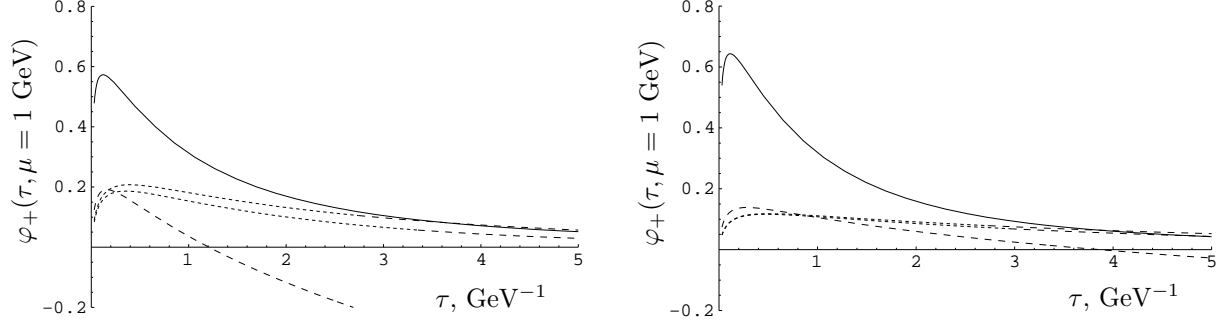


Figure 6: Perturbative (solid curves) and nonperturbative (long dashes) contributions to the B-meson distribution amplitude  $\varphi_+(\tau, \mu = 1 \text{ GeV})$  calculated from the sum rule (32) to the NLO accuracy. In addition, the nonlocal condensate models (35), (36) of resummed nonperturbative contributions to the sum rule, cf. (37), are shown by short dashes. The continuum threshold is chosen to be  $\omega_0 = 1 \text{ GeV}$  and two values of the Borel parameter are used:  $M = 0.3 \text{ GeV}$  (left panel) and  $M = 0.6 \text{ GeV}$  (right panel). The value of the decay constant  $F(\mu)$  appearing on the l.h.s. of (32) is substituted by the corresponding sum rule (15).

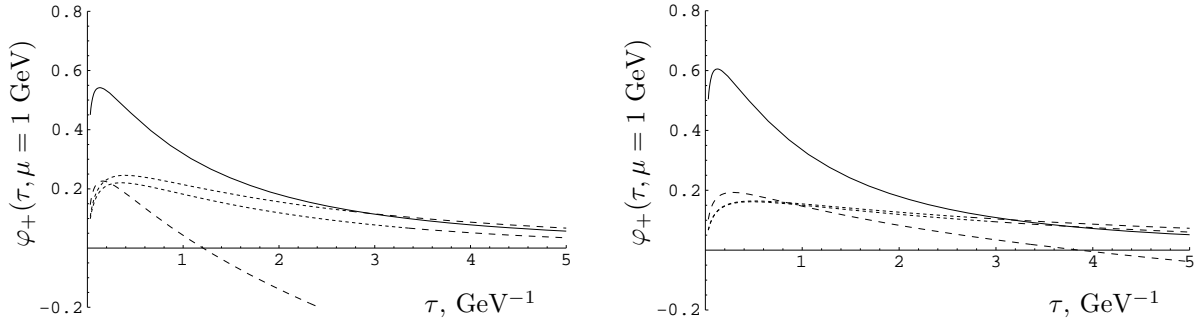


Figure 7: Same as in Fig. 6, with a different value of the continuum threshold  $\omega_0 = 0.8 \text{ GeV}$ .

parameter. A rough estimate for the nonperturbative contribution to  $\lambda_B^{-1}$  in the strict OPE-based approach is, therefore, given by the integral over the region of small  $\tau$  where the correction is still positive, that is up to the crossing point with the zero axis. In order to get an estimate of a possible nonperturbative contribution from large distances we use the concept of a nonlocal quark condensate introduced in [31] and later used rather extensively in QCD sum rule calculations of the distribution amplitudes of light mesons by the Dubna group [32, 33]. The same approach was taken up in [14].

The nonlocal quark condensate presents a model for a partial resummation of the OPE to all orders in terms of the vacuum expectation value of the single nonlocal operator

$$\langle 0 | \bar{q}(x)[x, 0]q(0) | 0 \rangle = \langle \bar{q}q \rangle \int_0^\infty d\nu e^{\nu x^2/4} f(\nu). \quad (33)$$

The first two moments of  $f(\nu)$  are fixed by the OPE:

$$\int_0^\infty d\nu f(\nu) = 1, \quad \int_0^\infty d\nu \nu f(\nu) = \frac{1}{4} m_0^2, \quad (34)$$

and in addition one requires that the correlation function (33) decreases exponentially at large Euclidian separations  $x^2 \rightarrow -\infty$ . The two simplest choices are

$$\text{Model I : } f(\nu) = \delta(\nu - m_0^2/4) \quad [31] \quad (35)$$

corresponding to the Gaussian large-distance behavior  $\sim \exp[-|x^2|m_0^2/16]$  and

$$\text{Model II : } f(\nu) = \frac{\lambda^{p-2}}{\Gamma(p-2)} \nu^{1-p} e^{-\lambda/\nu}, \quad p = 3 + \frac{4\lambda}{m_0^2} \quad [28] \quad (36)$$

corresponding to  $\langle 0|\bar{q}(x)q(0)|0\rangle \sim \exp[-\lambda\sqrt{-x^2}]$ . Here  $\lambda$  is a parameter with physical meaning of the vacuum quark correlation length. In this work we take  $\lambda = 400$  MeV as a representative number, cf [33, 34]. We will see that sensitivity of the sum rules to the shape of  $f(\nu)$  is in fact small; the major shortcoming of this approach is rather that other condensates (e.g. the nonlocal quark-antiquark-gluon condensate) are not included and there exists no parameter that would justify such a truncation.

Nonlocality of the quark condensate is easy to implement within our sum rules and it amounts to a simple substitution in Eq. (32) (cf. [14])

$$\begin{aligned} & -\frac{1}{2}\langle\bar{q}q\rangle S(\tau, M, \mu) \left\{ 1 + \mathcal{O}(\alpha_s) - \frac{1}{16} \frac{m_0^2}{M^2} (1 + 2\tau M) \right\} \\ & \rightarrow -\frac{1}{2}\langle\bar{q}q\rangle S(\tau, M, \mu) \int_0^\infty d\nu f(\nu) e^{-\nu(1+2\tau M)/(4M^2)}. \end{aligned} \quad (37)$$

Note that the mixed condensate contribution is now included as a part of the nonlocal condensate and we neglect the  $\alpha_s$ -correction to the (local) quark condensate (but retain the Sudakov exponent). The results are shown in Fig. 6 and Fig. 7 by short dashes; the lower and the higher of the curves correspond to the choice in (35) and (36), respectively. The corresponding results for  $\lambda_B^{-1}$  are, for  $\mu = 1$  GeV:

$$\begin{aligned} \omega_0 = 1 \text{ GeV}, \quad M = 0.6 \text{ GeV} : \quad \lambda_B^{-1} &= 1.23 + \begin{Bmatrix} 0.26 \\ 0.60 \\ 0.83 \end{Bmatrix} = 1.95 \pm 0.23 \text{ GeV}^{-1}, \\ \omega_0 = 1 \text{ GeV}, \quad M = 0.3 \text{ GeV} : \quad \lambda_B^{-1} &= 1.32 + \begin{Bmatrix} 0.13 \\ 0.54 \\ 0.88 \end{Bmatrix} = 2.03 \pm 0.29 \text{ GeV}^{-1}, \\ \omega_0 = 0.8 \text{ GeV}, \quad M = 0.6 \text{ GeV} : \quad \lambda_B^{-1} &= 1.36 + \begin{Bmatrix} 0.35 \\ 0.84 \\ 1.16 \end{Bmatrix} = 2.36 \pm 0.33 \text{ GeV}^{-1}, \\ \omega_0 = 0.8 \text{ GeV}, \quad M = 0.3 \text{ GeV} : \quad \lambda_B^{-1} &= 1.39 + \begin{Bmatrix} 0.15 \\ 0.64 \\ 1.05 \end{Bmatrix} = 2.24 \pm 0.35 \text{ GeV}^{-1}, \end{aligned} \quad (38)$$

where the first number gives the perturbative contribution (the difference to (22) is due to the different value used for  $F(\mu)$ ) and the three numbers under the brace correspond

to three different estimates for the nonperturbative contribution: 1) quark and mixed condensate contribution to (32) restricted to the positivity region, 2) nonlocal condensate model I (35) and 3) nonlocal condensate model II (36). The first (upper) number should be considered as an estimate of the nonperturbative correction from below while the difference between the two lower ones characterizes the uncertainty in the choice of the parametrization of the nonlocal condensate. We take the average between the two models as our central value, and one half of the difference between this central value and the first (upper) number, coming from local OPE, as an estimate of the overall uncertainty of the result. In other words, we ascribe 50% uncertainty to the extrapolation of the nonperturbative contribution to large distances as suggested by the nonlocal condensate model, which is rather conservative, cf. [32]. From the numbers in Eq. (38) we obtain the final result

$$\lambda_B^{-1}(\mu = 1 \text{ GeV}) = 2.15 \pm 0.5 \text{ GeV}^{-1} \quad (39)$$

or

$$\lambda_B(\mu = 1 \text{ GeV}) = 460 \pm 110 \text{ MeV}. \quad (40)$$

Our value of  $\lambda_B$  is somewhat larger than the number accepted in [35, 6]  $\lambda_B = 0.35 \pm 0.15 \text{ GeV}$ , although consistent with it within errors. It is also consistent with the rough estimate  $\lambda_B \simeq 0.6 \text{ GeV}$  derived in [15].

As follows from (5) and (6), the scale dependence of  $\lambda_B$  also involves the first logarithmic moment of the distribution amplitude [16, 12]

$$\lambda_B^{-1}(\mu) = \left[ 1 + \frac{\alpha_s C_F}{2\pi} \ln \frac{\mu}{\mu_0} \right] \lambda_B^{-1}(\mu_0) - \frac{\alpha_s C_F}{\pi} \ln \frac{\mu}{\mu_0} \int_0^\infty \frac{dk}{k} \ln \frac{\mu_0}{k} \phi_+(k, \mu_0), \quad (41)$$

where  $(\alpha_s C_F/\pi) \ln(\mu/\mu_0) < 1$ . We define

$$\sigma_B(\mu) = \lambda_B(\mu) \int_0^\infty \frac{dk}{k} \ln \frac{\mu}{k} \phi_+(k, \mu) = \lambda_B(\mu) \int_0^\infty d\tau \ln(\tau \mu e^{\gamma_E}) \varphi_+(\tau, \mu) \quad (42)$$

and calculate  $\sigma_B(1 \text{ GeV})$  from the QCD sum rule (32) repeating the same procedure as explained above for  $\lambda_B$ . Without going into details we simply quote the final result

$$\sigma_B(\mu = 1 \text{ GeV}) = 1.4 \pm 0.4. \quad (43)$$

Note that  $\sigma_B(\mu)$  defines the average value of  $\ln \mu/k$  in the integral for the first inverse moment  $\lambda_B^{-1}$ , so that the number in (43) implies that main contribution to  $\lambda_B^{-1}$  comes from momenta  $\sim 250 \text{ MeV}$ . With this value for  $\sigma_B$ , the two contributions  $\mathcal{O}(\alpha_s)$  in (41) tend to cancel each other to a large extent, so that the remaining scale dependence of  $\lambda_B^{-1}$  is weak.

A simple model of the distribution amplitude  $\phi_+(k, \mu)$  with given values of the parameters  $\lambda_B$  and  $\sigma_B$  and correct asymptotic behavior can be chosen as

$$\phi_+(k, \mu = 1 \text{ GeV}) = \frac{4\lambda_B^{-1}}{\pi} \frac{k}{k^2 + 1} \left[ \frac{1}{k^2 + 1} - \frac{2(\sigma_B - 1)}{\pi^2} \ln k \right] \quad (44)$$

( $k$  in units of GeV). Using the values of  $\lambda_B$  and  $\sigma_B$  in (39), (43) one obtains the distribution shown in Fig. 8 by the solid curve. For comparison, we also show in this plot a typical

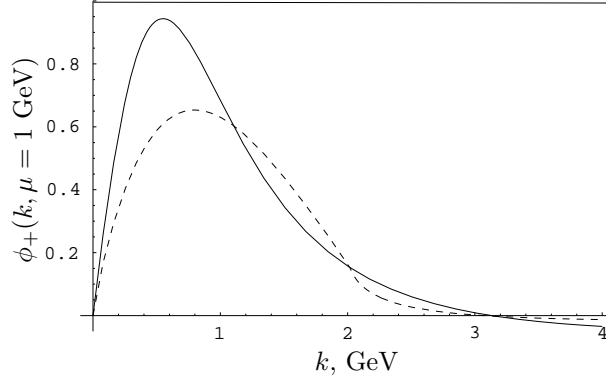


Figure 8: A QCD model for the B-meson distribution amplitude (44) (solid curve) compared with the perturbative sum rule prediction (16) with  $M=0.45$  GeV,  $\omega_0 = 1$  GeV (dashed curve).

distribution amplitude obtained from QCD sum rules in perturbation theory (16) in Sec. 3, cf. Fig. 3. The effect of nonperturbative corrections is to shift the distribution towards softer momenta, which is natural. One minor drawback of such a parametrization is that the set of parameters  $\lambda_B$  and  $\sigma_B$  is not closed under renormalization. In view of a very limited range of scales that are interesting for B-decay phenomenology this seems to be not a problem, however.

To summarize, in this paper we have derived QCD sum rules for the B-meson distribution amplitude (1) and, in particular, obtained an estimate of its first inverse moment  $\lambda_B^{-1}$  (39) and the parameter  $\sigma_B$  (42) that characterizes the shape of the distribution, see (43). A simple model is suggested (44) that incorporates all existing constraints. We believe that our estimates are interesting for the studies of the heavy quark limit in exclusive B-decays and can be used in a broad context. Concrete applications go beyond the task of this work.

## Acknowledgements

V.B. is grateful to the Laboratoire de Physique Théorique, Université Paris XI, for hospitality during his sabbatical stay in Orsay and CNRS for the financial support. Work of D.I. is partially supported by Alexander von Humboldt Foundation and by grants RFBR 02-02-17884 and INTAS 00-00679.

## References

- [1] A. Szczepaniak, E. M. Henley and S. J. Brodsky, Phys. Lett. B **243** (1990) 287.
- [2] V.L. Chernyak and A.R. Zhitnitsky, JETP Lett. **25** (1977) 510; Yad. Fiz. **31** (1980) 1053; V.L. Chernyak, V.G. Serbo and A.R. Zhitnitsky, JETP Lett. **26** (1977) 594; Sov. J. Nucl. Phys. **31** (1980) 552.



- [3] G.P. Lepage and S.J. Brodsky, Phys. Lett. B **87** (1979) 359; Phys. Rev. Lett. **43** (1979) 545,1625 (E); Phys. Rev. D **22** (1980) 2157; S.J. Brodsky, G.P. Lepage and A.A. Zaidi, Phys. Rev. **D23** (1981) 1152.
- [4] A.V. Efremov and A.V. Radyushkin, Phys. Lett. B **94** (1980) 245; Teor. Mat. Fiz. **42** (1980) 147.
- [5] G. P. Korchemsky, D. Pirjol and T. M. Yan, Phys. Rev. D **61** (2000) 114510.
- [6] S. Descotes-Genon and C. T. Sachrajda, Nucl. Phys. B **650** (2003) 356.
- [7] E. Lunghi, D. Pirjol and D. Wyler, Nucl. Phys. B **649** (2003) 349;  
S. W. Bosch, R. J. Hill, B. O. Lange and M. Neubert, Phys. Rev. D **67** (2003) 094014.
- [8] S. W. Bosch and G. Buchalla, JHEP **0208** (2002) 054.
- [9] S. Descotes-Genon and C. T. Sachrajda, Phys. Lett. B **557** (2003) 213.
- [10] E. Bagan, P. Ball and V. M. Braun, Phys. Lett. B **417** (1998) 154; J. Charles, A. Le Yaouanc, L. Oliver, O. Pene and J. C. Raynal, Phys. Rev. D **60** (1999) 014001; M. Beneke and T. Feldmann, Nucl. Phys. B **592** (2001) 3; M. Beneke, A. P. Chapovsky, M. Diehl and T. Feldmann, Nucl. Phys. B **643** (2002) 431; C. W. Bauer, D. Pirjol and I. W. Stewart, Phys. Rev. D **67** (2003) 071502; M. Beneke and T. Feldmann, arXiv:hep-ph/0308303.
- [11] A. Ali, V. M. Braun and H. Simma, Z. Phys. C **63** (1994) 437.
- [12] P. Ball, arXiv:hep-ph/0308249.
- [13] V. L. Chernyak and A. R. Zhitnitsky, Nucl. Phys. B **201** (1982) 492; Phys. Rept. **112** (1984) 173.
- [14] A. G. Grozin and M. Neubert, Phys. Rev. D **55** (1997) 272.
- [15] P. Ball and E. Kou, JHEP **0304** (2003) 029.
- [16] B. O. Lange and M. Neubert, arXiv:hep-ph/0303082.
- [17] M. Neubert, Phys. Rept. **245** (1994) 259.
- [18] I. A. Korchemskaya and G. P. Korchemsky, Phys. Lett. B **287** (1992) 169;  
G. P. Korchemsky and G. Marchesini, Nucl. Phys. B **406** (1993) 225.
- [19] I. I. Balitsky and V. M. Braun, Nucl. Phys. B **311** (1989) 541.
- [20] H. Kawamura, J. Kodaira, C. F. Qiao and K. Tanaka, Phys. Lett. B **523** (2001) 111  
[Erratum-ibid. B **536** (2002) 344].
- [21] M. A. Shifman, A. I. Vainshtein and V. I. Zakharov, Nucl. Phys. B **147** (1979) 385;  
*ibid.* B **147** (1979) 448.

- [22] E. Bagan, P. Ball, V. M. Braun and H. G. Dosch, Phys. Lett. B **278** (1992) 457.
- [23] M. Neubert, Phys. Rev. D **45** (1992) 2451.
- [24] P. Ball and V. M. Braun, Phys. Rev. D **49** (1994) 2472.
- [25] D. J. Broadhurst and A. G. Grozin, Phys. Lett. B **274** (1992) 421.
- [26] I. I. Balitsky, A. V. Kolesnichenko and A. V. Yung, Yad. Fiz. **41** (1985) 282.
- [27] V. M. Belyaev and B. L. Ioffe, Nucl. Phys. B **310** (1988) 548; Int. J. Mod. Phys. A **6** (1991) 1533; B. L. Ioffe and A. Khodjamirian, Phys. Rev. D **51** (1995) 3373.
- [28] V.M. Braun, P. Gornicki and L. Mankiewicz, Phys. Rev. D **51** (1995) 6036.
- [29] A. G. Grozin and G. P. Korchemsky, Phys. Rev. D **53** (1996) 1378.
- [30] J.G.M.Gatheral, Phys. Lett. 133B (1983) 90; J.Frenkel and J.C.Taylor, Nucl. Phys. B246 (1984) 231.
- [31] S. V. Mikhailov and A. V. Radyushkin, JETP Lett. **43** (1986) 712 [Pisma Zh. Eksp. Teor. Fiz. **43** (1986) 551]; Sov. J. Nucl. Phys. **49** (1989) 494 [Yad. Fiz. **49** (1988) 794].
- [32] A. P. Bakulev and A. V. Radyushkin, Phys. Lett. B **271** (1991) 223; S. V. Mikhailov and A. V. Radyushkin, Phys. Rev. D **45** (1992) 1754; A. P. Bakulev and S. V. Mikhailov, Z. Phys. C **68** (1995) 451; Phys. Lett. B **436** (1998) 351.
- [33] A. P. Bakulev and S. V. Mikhailov, Phys. Rev. D **65** (2002) 114511.
- [34] A. V. Radyushkin, Phys. Lett. B **271** (1991) 218.
- [35] M. Beneke, G. Buchalla, M. Neubert and C. T. Sachrajda, Phys. Rev. Lett. **83** (1999) 1914; Nucl. Phys. B **591** (2000) 313.